

# STRUCTURAL DAMAGE DETECTION USING THE HOLDER EXPONENT

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## ABSTRACT

This paper implements a damage detection strategy that identifies damage sensitive features associated with non-linearities. Some non-linearities result from discontinuities introduced into the data by certain types of damage. These discontinuities may also result from noise in the measured dynamic response data or can be caused by random excitation of the system. The Holder Exponent, which is a measure of the degree to which a signal is differentiable, is used to detect the discontinuities. By studying the Holder Exponent as a function of time, a statistical model is developed that classifies changes in the Holder Exponent that are associated with damage-induced discontinuities. The results show that for certain cases, the Holder Exponent is an effective technique to detect damage.

## NOMENCLATURE

$D(t)$	Damage event
$H(t)$	Holder exponent (HE)
$\psi(\tau, s)$	Continuous Wavelet Transform (CWT)
$\psi$	Mother Wavelet (Morlet Wavelet)
$a$	Order of Morlet Wavelet
$\alpha$	Scale factor for Morlet Wavelet
$f$	Frequency
$m$	Log slope of the magnitude of the frequency spectrum
$n$	Scalar multiplier
$s$	Scaling term for CWT
$\sigma$	Standard deviation
$t$	Time
$\tau$	Translation term for CWT
$x(t)$	Acceleration signal
$\overline{X(t)}$	Moving mean of HE signal

## 1. INTRODUCTION

Engineering systems require damage monitoring for purposes of safety and performance. Structures that have undergone geometric or material changes that adversely affect their performance are assumed to have damage features that can be extracted from dynamic response measurements. The process of detecting these damage features is known as structural health monitoring (SHM).

SHM involves monitoring the dynamic response of the system over time and extracting damage features using signal processing techniques. Long-term applications include monitoring the ability of structures components, such as a joint, to perform their intended function. SHM also provides the means to diagnose the condition of a structure following a traumatic event. For instance, after an earthquake, SHM systems can provide information about the integrity of a structure.

Feature extraction is the process of identifying damage-sensitive parameters from the dynamic response of the system. Some feature extraction techniques include identifying the response amplitude, mode shapes, resonant frequencies, or quantities derived from these parameters such as the dynamic flexibility matrix [1]. These techniques and other methods of damage detection assume that the system is linear before and after the damage event. However, damage such as the opening and closing of cracks on a bridge or loose bolts in a joint introduces non-linearities to the system. Robertson hypothesized that the damage associated with nonlinearities— especially at onset — causes discontinuities in the acceleration response of the system [2].

The proposed method to extract damage features takes advantage of the non-linearities by detecting the discontinuities in the acceleration-time signal using the Holder Exponent (HE). The Holder Exponent is a measure of the regularity of a signal. A discontinuity causes a signal to be irregular, which causes a change in the Holder Exponent. It is the purpose of this project to examine the feasibility of monitoring the changes in the Holder Exponent as a damage detection strategy.

The following sections describe the method of damage detection using the Holder Exponent. This procedure is applied to acceleration signals obtained from a variety of sources, including a numerical model with simulated damage and three structures with different localized damage. The damage features are extracted from the signals by performing a Wavelet Transform and subsequently calculating the Holder Exponent from the transform. Finally, a statistical process is described to correlate the changes in the Holder Exponent to discontinuities associated with damage.

## 2. METHODOLOGY

There are three primary steps in the damage detection process. They are: data acquisition, feature extraction, and statistical modeling.

### 2.1 Data Acquisition

Data acquisition is the process of selecting the types, location, and quantity of sensors, and subsequently putting these sensors to use. For this project, three experimental models are investigated: a cantilever beam, a five degrees-of-freedom (DOF) structure, and a three-story structure. For each structure, acceleration is measured using single-axis piezoelectric accelerometers. Both the cantilever beam and the five DOF setup use accelerometers with nominal sensitivities of 10 mV/g and dynamic range from 1 to 10000 Hz. The three-story structure uses accelerometers with sensitivities of 1 V/g and bandwidth from 1 to 2000 Hz.

The signals from the accelerometers are processed using various commercial analog-to-digital (A/D) converters and recorded by data acquisition software. Resolutions on the A/D converters vary between 13 and 24-bit and sampling rates are chosen so that high frequency content (up to 2 kHz) can be detected.

### 2.2 Feature Extraction

Time signals are converted to both time and frequency domains using the Continuous Wavelet Transform (CWT). The CWT,  $\Psi(\tau, s)$ , is defined as:

$$\Psi(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \psi^* \left( \frac{t - \tau}{s} \right) dt \quad (2.1)$$

where  $x(t)$  is the acceleration-time signal,  $\tau$  is the translation parameter, and  $s$  is the scale parameter. The scale parameter  $s$  dilates the transforming function (or Mother Wavelet)  $\psi$ . This parameter is a variable term and allows for multi-resolution analysis of the signal. The CWT of the signal is calculated at each value of  $s$  that is translated by time  $\tau$ . At high frequencies,  $s$  is small; therefore, good time resolution is obtained, but with poor frequency resolution. When  $s$  is large, there is good frequency resolution but poor time resolution. For a complete discussion of the Wavelet Transform, see *The Wavelet Tutorial* by Robi Polikar [3].

There are a variety of Mother Wavelets to choose from. For this discussion, the Morlet Wavelet is chosen because it is applicable to a variety of signals. The Morlet Wavelet,  $\psi^*$ , is a sinusoid within a Gaussian envelope and is given by

$$\psi^* = e^{iat} e^{\frac{-t^2}{2\alpha}} \quad (2.2)$$

where  $t$  is time,  $a$  is the order, and  $\alpha$  is the scale factor. It is determined that an order of 16 and a scale factor of 4 generally provide good resolution of the time signals. The result of the CWT is a function of  $\tau$  and  $s$ , which are strongly related to time and frequency, respectively. Thus, time and

frequency content can be retrieved from these values and plotted on an image contour map called a scalogram. The scalogram is the magnitude response of the CWT in the time and frequency domain.

Using the CWT is advantageous for two reasons. First, the CWT contains the original signal's frequency spectrum for each instant in time. The Holder Exponent can then be extracted from each frequency spectrum and provides a measure of signal regularity as a function of time. If the Holder Exponent were extracted from a Fourier Transform (FT), then it would give only the global regularity of the signal and would not detect local discontinuities. The second advantage of the CWT is the ability to perform multi-resolution analysis. This ability is to be compared with the Short Time Fourier Transform (STFT), for which the resolution is constant for all time and frequencies. Multi-resolution analysis allows the CWT to both pinpoint high frequency content in time and to separate such content from low frequency information. Both of these aspects are crucial in obtaining the Holder Exponent.

The damage sensitive feature that is to be extracted is the Holder Exponent. The Holder Exponent is a measure of the regularity, or the differentiability, of a signal. During a damage event, such as crack initiation, a step or impulse is introduced into the acceleration-time signal. This step or impulse causes the signal to be not differentiable at that instant. The differentiability of a signal is ascertained by first calculating the asymptotic decay of the frequency spectrum [2]. The Holder Exponent,  $H(t)$ , is then defined as:

$$H(t) = \frac{(-m(t) - 1)}{2} \quad (2.3)$$

where  $t$  is time and  $m$  is the slope of the logarithmic decay of the magnitude of the frequency spectrum. In a structure being excited by ambient forces, such as those forces resulting from ground motion during an earthquake, there is greatest energy at relatively low frequencies. The energy is at the frequencies of the input, which typically range between 0 and 10 Hz for an earthquake, or at the first few resonant frequencies of the structure. High frequency content like noise or other random inputs is generally at a relatively small energy level and is attenuated by the structure. When a signal from a healthy or undamaged structure is analyzed, decay in the magnitude of the frequency spectrum from low to high frequencies is seen. This decay can be fit well on a log-log plot with a line of negative slope. From [2.3], the Holder Exponent is positive when there is slope less than -1. A discontinuity in the signal, presumably from a damage event, introduces energy content at high frequencies into the frequency spectrum, causing the slope to approach zero or a positive value. By using the CWT, there is excellent time resolution at high frequencies, capturing high frequency content at a precise instant in time. Therefore, changes toward zero in the slope of the frequency spectrum happen at a well-defined point in time. This change results in a dip in the Holder Exponent when plotted versus time.

### 2.3 Statistical Modeling

Statistical modeling is used to characterize the dip in the Holder Exponent as a damage event. In order to classify a drop in the Holder Exponent as a damage event, there needs to be a healthy signal for comparison. The healthy signal is used to *train* the statistical classification procedure. Once boundaries based on statistical parameters from the healthy signal have been established, a questionable signal can be analyzed to assess whether the changes in the Holder Exponent are the result of damage.

The statistical parameters to be extracted from the Holder Exponent time series are mean and standard deviation. First, a global standard deviation is calculated from the Holder Exponents associated with the healthy structure. This parameter establishes a measure of the normal variability of the Holder Exponent from an undamaged structure. A moving mean is then calculated from the questionable data by taking the mean on a window centered about a data point, then shifting the window to the next point. These two parameters are used to construct control limits for the statistical model. Control limits are boundaries that are set by the probabilistic distribution of the data. For a healthy data set, the control limits define the variation of the healthy values, or the normal condition of a structure. Damage occurs when the Holder Exponents of the questionable signals fall outside the lower control limit from the healthy data. This relationship is defined as:

$$D(t) \leq \overline{X(t)} - n\sigma \quad (2.4)$$

where  $D$  is the occurrence of damage,  $X$  is the moving mean of the questionable HE data,  $n$  is a multiplier,  $\sigma$  is the standard deviation of the healthy HE data, and  $t$  is time. The value  $n$  is determined by first examining a simple system such as the cantilever beam. A value of  $n$  that works for the beam model is then applied to the control limits of more complex models: the five DOF system and the three-story structure.

### 3. APPLICATIONS

The sensitivity of the Holder Exponent in detecting damage is studied by testing the method in four situations, including both analytical and physical models of damaged structures. The studies try to address the following questions:

1. Is the method applicable to different types of damage?
2. How sensitive is the method to changes in damage condition?
3. Can the method detect damage in the presence of noise or random excitation?
4. How does sensor position affect the ability to detect damage?

The first case involved analyzing signals from a numerical model derived from a damaged structure. This model is a preliminary study on the sensitivity of the Holder Exponent. In the second case, a simple experiment is used to understand the response of the Holder Exponent in a physical model. A cantilever beam is given an initial displacement and collides against another structure as it vibrates. The third case is a five DOF spring and mass

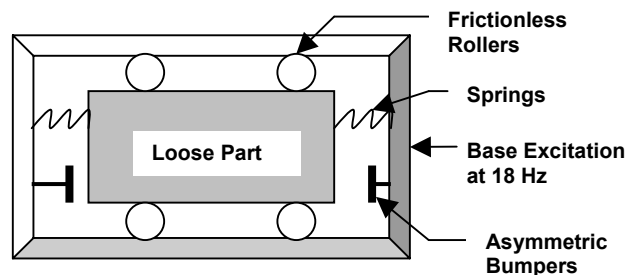
system with bumpers between two of the masses. A shaker drives the system with both a periodic and random signal. Damage occurs when the bumpers impact one another, which simulates loose parts in a housing. The purpose of having two different input waveforms is to determine whether the Holder Exponent can detect damage during periodic vibrations or, more realistically, when random vibrations are present. Placing sensors away from the bumpers and softening the bumper impact points also test the sensitivity of the detection method.

Lastly, a three-story frame structure is analyzed to simulate a building that incurs damage during an earthquake. In this application, damage is defined as the sudden loss of bolt pretension at a joint. In the three-story structure, the floor plates are attached to the columns by bolted joints. At one joint is a piezoelectric stack actuator in place of the washer on a bolt. While the system is excited, the stack deactivates, causing a step change in the tension at the joint. Like the five DOF system, accelerometers are placed at different locations and the excitation force is both periodic and random.

### 3.1 Numerical Simulation

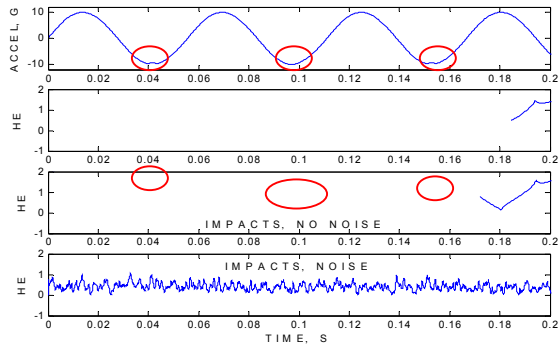
Robertson determined that the Holder Exponent is an effective method to detect loose parts in a structure [2]. Specifically, Robertson analyzed data from a structure with a loose internal part. The structure was driven by a sinusoidal input. The Holder Exponent detected impacts in the structure that were not obvious in the time signal or the frequency spectrum.

To recreate the signal for purposes of further analysis, a numerical model was built based on the schematic shown in Figure 3.1. The model was built using the *Simulink* toolbox in *Matlab*. To simulate the impacts of the bumpers, impulses were used in the model. The impulses introduced discontinuities in the time response and the Holder Exponent was used to detect the discontinuities. To make the model more realistic, noise was added to the sinusoidal input signal.



**Figure 3.1.** Schematic model of a structure with a loose part.

The results of the numerical simulation show two important points. When there are impacts in the system without noise, visible changes in the HE time response can be detected. However, when noise is present in the signal, there are no noticeable changes in the HE signal at the impact points. Figure 3.2 compares the HE signals without impacts, with impacts and no noise, and impacts with noise. The first plot is the acceleration-time history with the impacts circled in red.

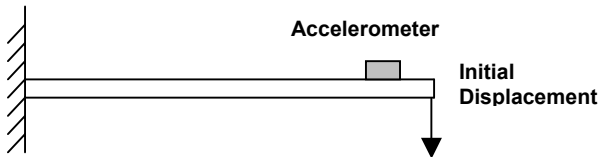


**Figure 3.2.** Comparison of different Holder Exponent signals from the numerical simulation.

From Figure 3.2, the impact Holder Exponent signal is similar to the baseline (no impacts) signal, except at the impact points, where there is significant change. However, noise in the last signal changes the shape of the data and also masks the changes of the Holder Exponent caused by impacts.

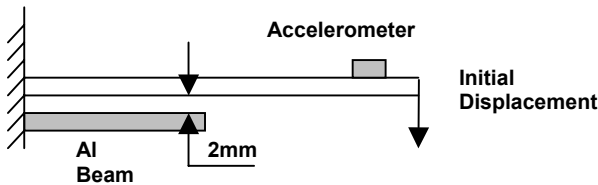
### 3.2 Cantilever Beam

A simple model was developed to understand the sensitivity of the Holder Exponent to detect damage features. Initially, a plexiglass beam with dimensions of 700 x 50 x 9.5 mm (L x W x H) was grounded at one end to a stationary structure. Figure 3.3 shows the simple cantilever beam setup. The beam was given an initial displacement and an accelerometer placed 14.5 mm from the free end measured the time response of the beam.



**Figure 3.3.** Baseline cantilever beam setup.

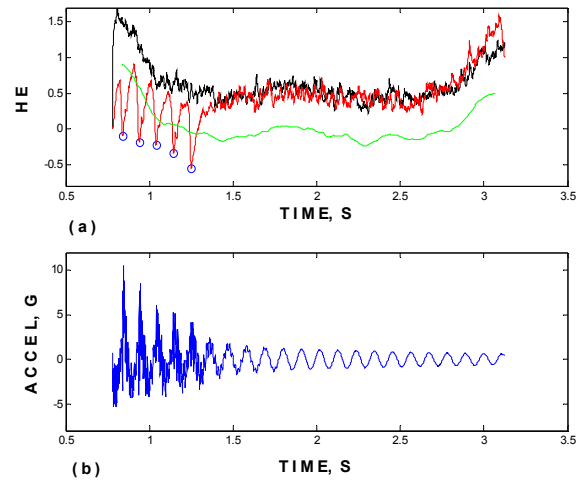
Next, to simulate damage analogous to a crack opening and closing, an aluminum beam measuring 350 x 50 x 3.6 mm was placed 2 mm below the plexiglass beam, as shown in Figure 3.4. Again, an initial displacement was applied to the plexiglass beam and the time response was measured with the accelerometer. Initially, the amplitude of vibration of the plexiglass beam was sufficiently large so that it came into contact with the aluminum beam. When the response of the beam died down to the point that the beam no longer collided with the aluminum, the beam response was similar to that of the first case.



**Figure 3.4.** Cantilever beam test with contact.

Acceleration was measured by a piezoelectric accelerometer with an integrated charge amplifier. The signal was digitized using a Dactron Photon 24-bit data acquisition system and recorded by the RT-Pro data acquisition software.

Results from the cantilever beam test show promise for using the Holder Exponent as a damage sensitive feature. During instances of impact, the Holder Exponents of the acceleration-time signal dip well outside the range of the baseline signal, as shown in figure 3.5a. By using a  $n$  value of 2 in equation [2.4] (or 2 standard deviations from the mean of the questionable signal), the statistical process described in Section 2.3 was implemented to detect the dips in the HE signal that are associated with damage. The statistical process successfully classified all significant dips in the HE, shown as blue circles in Figure 3.5a.

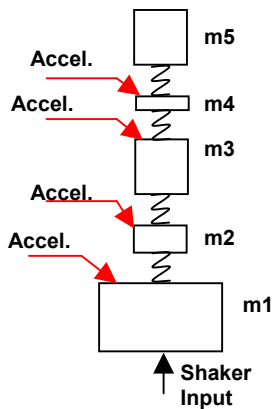


**Figure 3.5.** (a) HE signals from cantilever beam tests (Black – baseline, Red – questionable, Green – control limit, Blue circles – detected damage events). (b) Acceleration-time signal from test with contact.

Although the Holder Exponent successfully detected discontinuities associated with damage, Figure 3.5b shows that the discontinuities can be seen in the acceleration signal as well. Obvious discontinuities in the acceleration signal bring the usefulness of the Holder Exponent into question in this case. However, the results from this test are important because they provide the statistical parameters to analyze other structures.

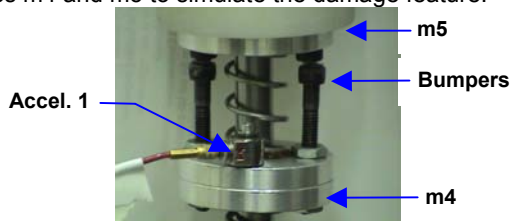
### 3.3 Five Degrees-of-Freedom Structure

The next test structure was a five degrees-of-freedom (DOF) structure. The structure was composed of five unequal masses and four springs of varying stiffness. Piezoelectric accelerometers were placed on four masses, as shown in Figure 3.6.



**Figure 3.6.** Five DOF system experimental setup.

The five DOF system was excited by a VTS single axis shaker connected to mass m1. Three different input waveforms were applied: a sine wave at 10 Hz, a sine wave at 18 Hz, and a shaped random with frequency content between 10 and 40 Hz. The waveforms were generated by the RT-Pro data acquisition software and sent to a Techtron 5530 Power Amplifier used to drive the shaker. Bumpers (shown in Figure 3.7) were placed 2 mm apart between masses m4 and m5 to simulate the damage feature.



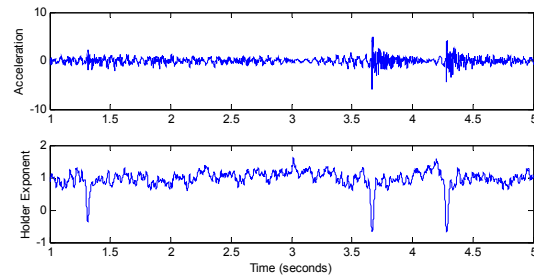
**Figure 3.7.** Bumpers on five DOF system.

For each input waveform, a baseline run was performed in which the bumpers did not contact. On the contact runs, the gain of the amplifier was set high enough so that the bumpers touched at the highest amplitude. This contact was identified by audible recognition. The contact runs were performed with three bumper configurations: metal-to-metal, electrical tape to metal, and hot glue to metal. Data were digitized using the Dactron Spectrabook 24-bit analog to digital converter and RT-Pro recorded the time response.

In the five DOF system, changes corresponding to impacts could be seen in the Holder Exponent of the acceleration signal from the mass closest to the bumper impacts in all test cases. The acceleration signals of the other masses had mixed results depending on the input. Analysis of the Holder Exponent at different locations relative to the origin of the discontinuity is further discussed in Section 4.1 (Sensitivity to Sensor Location).

Very promising results came from a shaped random excitation from 10 to 40 Hz. The acceleration-time plot in Figure 3.8 clearly shows two discontinuities in the latter portion of the signal between 3.5 and 4.5 seconds. These discontinuities manifest themselves as dips in the Holder Exponent quite well. Also in the Holder Exponent is a dip between 1.0 and 1.5 seconds. In the time signal there is no clearly visible discontinuity at this point. This signal contains

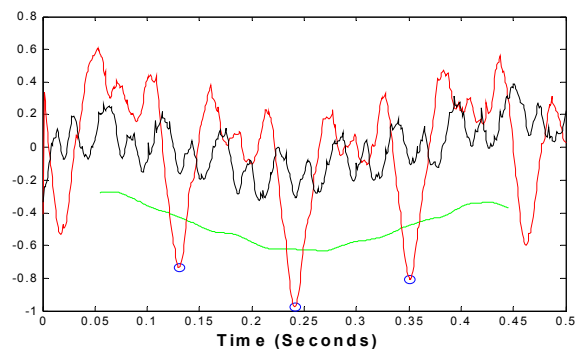
an excellent example of a damage event that the Holder Exponent can excel at detecting. Unfortunately, minor impact such as these were not easily reproduced in the five DOF system. This difficulty was a direct result of the need to audibly recognize impacts to ensure the occurrence of impacts during a test run.



**Figure 3.8.** Acceleration and Holder Exponent from first mass on the five DOF system.

The statistical control developed using the cantilever beam experiment was used to interrogate the Holder Exponent signals from the five DOF system. Much of the data from the five DOF system exhibited trends in the experimental signals that were not present in the baseline signals. This phenomenon was quite puzzling. These trends involved changes in the Holder Exponent that occurred on a time scale much larger than that of the changes that could be associated with damage. Though a formal investigation into the causes of these trends has not been conducted, this feature of the Holder Exponent may be attributed to two sources. First, these changes may be caused by variation in the random noise of the system. The second explanation requires considering the source of the Holder exponent itself. Recall that the Holder Exponent is calculated from the wavelet transform of the time-acceleration signal. If damage is considered an impulse of broad frequency content the damage will manifest itself in the scalogram as a peak with a broad base. This broad base results from the poor time resolution of the wavelet transform at low frequencies.

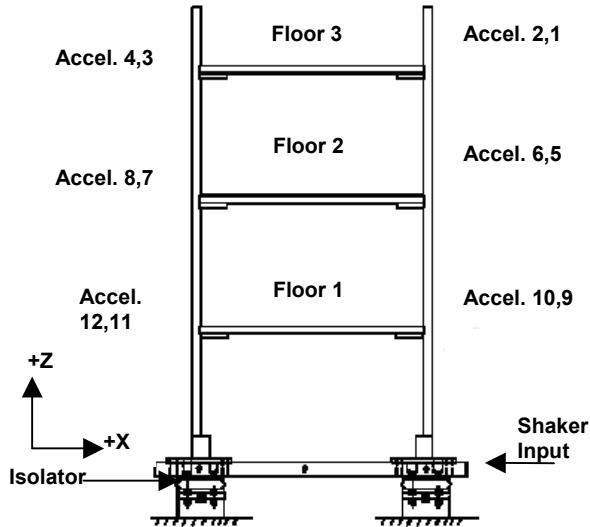
As described previously, the moving mean of the questionable signal was used to establish control limits. This procedure allows the control limit to tracks global changes in the Holder Exponent while still remaining sensitive to the damage events that occur on a shorter time scale. The application of the statistical control process for the five DOF system is shown in Figure 3.9.



**Figure 3.9.** Five DOF system excited at 10 Hz with impacts (Black – baseline, Red – questionable, Green – control limit, Blue circles – detected damage events).

### 3.4 Three-Story Frame Structure

The three-story frame structure was constructed of Unistrut columns and aluminum floor plates, shown in Figure 3.10. The structure was 0.762 x 0.610 x 1.55 m (L x W x H) and the floors were 0.470 m apart. At the base plate were four air mount vibration isolators that allow the structure to move freely in the horizontal directions. Four piezoelectric accelerometers were placed on each floor, located at each joint that connected the plates to the columns. The accelerometers were oriented to measure accelerations in the Z-direction. In Figure 3.10, the accelerometers are numbered such that the first number listed corresponds to the accelerometer at the front of the structure as drawn.



**Figure 3.10.** Three-story frame structure setup.

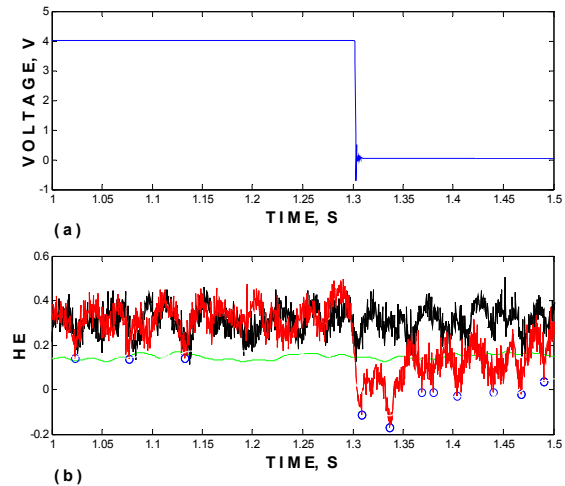
The loosening of a bolt was caused by a piezoelectric stack actuator attached to a bolt at a joint, as shown in Figure 3.11. Initially, the stack was activated by a voltage (~800V), expanding the stack. The nut was tightened to 16.9 N-m, as were all other nuts on the structure. While the system was being driven by either a periodic or a random force, the stack was deactivated, causing the stack to contract and relieving a portion of pretension in the bolt. Different voltage levels and signals to the stack were investigated. The voltage levels ranged from 200 to 1000 V; the level was directly proportional to the expansion (or stroke) of the stack. Step and square wave voltage signals caused the stack to expand and contract at specified intervals.



**Figure 3.11.** Piezoelectric stack actuator attached to a bolt.

Data acquisition for the three-story structure required the most hardware. The voltage to the stack actuator was controlled by the Stanford Research System Pulse Generator DG535 and the Wavetek 191 Pulse Generator. The Stanford System generated the single step voltage source and the Wavetek generated the square wave voltage signals. Input to the system was provided by the VTS single axis shaker with input signals generated by the Hewlett-Packard HP3566A data acquisition module. The signals were an 18 Hz sine excitation, a broadband random excitation from 0 to 800 Hz, and shaped (or band-limited) random excitation from 0 to 100 Hz. Signals from the accelerometers were amplified by the Techtron Power Amplifier and collected by the HP3566A with a 13-bit A/D converter. The HP3566A also collected signals from the Stanford and Wavetek pulse generators. Data analysis was performed by the HP data acquisition software.

The results from the three-story tests vary depending on the type of excitation. When the structure was driven by a sinusoidal input, the results showed that the Holder Exponent detected the loss of pretension in the bolt. The data was extracted from the accelerometer closest to the piezoelectric stack. Figure 3.12a shows the voltage to the piezoelectric stack where the drop in voltage corresponded to the loss of pretension in the bolt. Figure 3.12b shows the Holder Exponent of the questionable signal in red.



**Figure 3.12.** (a) Voltage to the piezoelectric stack. (b) HE signals from three-story with sine input (Black – baseline, Red – questionable, Green – control limit, Blue circles – detected damage events).

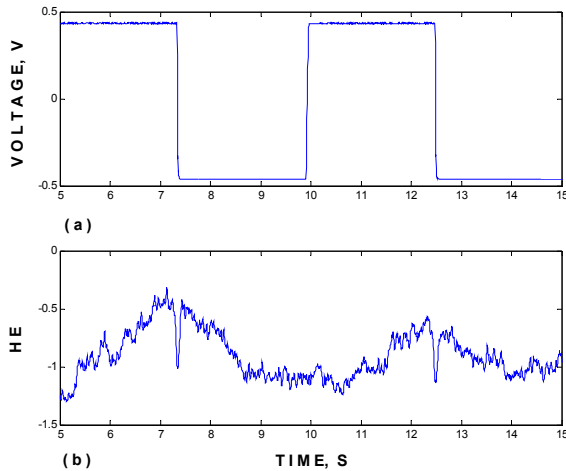
The statistical process control described in section 2.3 was applied to the data, shown by Figure 3.12b. The model flagged many changes in the Holder Exponent that were not related to a damage event. This failure of the statistical process control reinforces the fact that statistical modeling is application specific. However, there is a noticeable drop in the Holder Exponent; therefore, a different statistical process control should be able to detect the drop without false positives.

The next type of excitation into the three-story structure was a broadband random input that varied from 1 to 1000 Hz. In this environment, any changes in the Holder Exponent



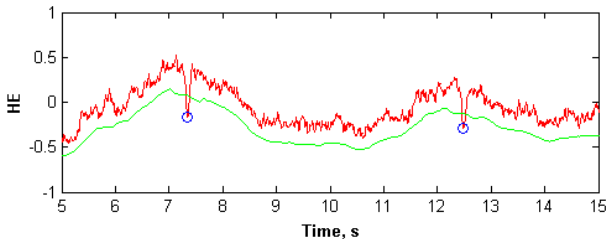
related to a damage event were not significant compared to other extraneous changes in the Holder Exponent. Initially, this lack of sensitivity was puzzling until the fundamentals of the Holder Exponent were reexamined. A discontinuity, or impulse, in the time domain produces broadband content in the frequency domain. When there is a broadband input signal, energy at a wide band of frequencies exists; thus, the energy from the discontinuity is masked. This masking results in a loss of sensitivity in the capability of the Holder Exponent.

Armed with the insight about the inability of the Holder Exponent to detect discontinuities in the presence of broadband frequencies, the three-story structure was revisited. By limiting the bandwidth of the random input from 0 to 100 Hz, it was hypothesized that the Holder Exponent would more easily accentuate the high frequency content resulting from a damage event. Figure 3.13a shows the voltage to the piezoelectric stack and Figure 3.13b shows the HE signal of the damage case in red. Any drop in the voltage applied to the piezoelectric stack corresponded to a loss of bolt tension. Figure 3.13b proves the hypothesis to be correct; significant dips in the Holder Exponent correlate exactly to the loss in tension. The shaped random input allowed the Holder Exponent to detect high frequencies produced by damage events.



**Figure 3.13.** (a) Voltage to the piezoelectric stack. (b) HE signals from three-story with shaped random input.

The same statistical process described previously was applied to the three-story structure data. Figure 3.14 shows the statistical model successfully detected the dips in the HE associated with the damage.



**Figure 3.14.** HE signal of three-structure test with shaped random input (Red – questionable, Green – control limit, Blue circles – detected damage events).

The findings from the three-story data have promising real world applications. For example, an earthquake typically has the majority of its energy in the frequency band of 0 to 10 Hz. Discontinuities in the acceleration signal of a structure caused by the loosening of a bolt introduce energy at much higher frequencies (above 100 Hz). In this case, the Holder Exponent is an effective identifier of damage events.

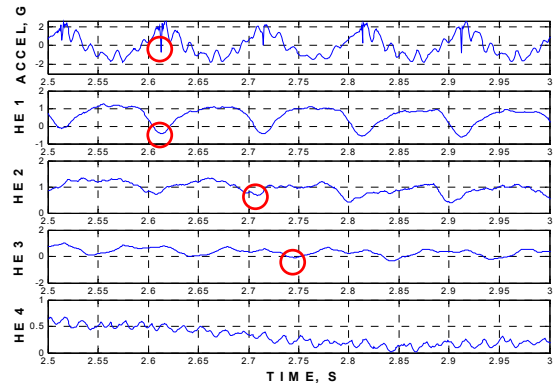
#### 4. SENSITIVITY

A study of the sensitivity of the Holder exponent under varied conditions was conducted using multiple structures. This study aimed to explore the effects of variation in the location of the accelerometer relative to the location of the damage point. Also of interest is the level of damage that needs to be present for the Holder Exponent to detect the discontinuity.

In the first attempt to characterize the sensitivity of the detection scheme by analyzing the Holder Exponent at varied locations, the five DOF was used. Figure 4.1 shows the acceleration-time history of accelerometer 1 followed by the Holder Exponent for each of the channels during sinusoidal excitation of the structure. To demonstrate the sensitivity of the method, a signal was chosen that showed the discontinuity in the acceleration time history. The Holder Exponent of the signal from the accelerometer closest to the damage clearly shows dips corresponding to the discontinuities induced by the bumper impact. In channels 2 and 3 the impacts are less clear, but still distinct. Channel four does not show distinct dips. In signals with less intense impacts, channels 2 and 3 did not produce a detectable result.

When the data resulting from a shaped random input were analyzed, the Holder Exponent lacked any sensitivity to the damage at a sensor location removed from the point of damage.

The limited sensitivity of the Holder Exponent in the five DOF system is important for real world applications. The purpose of the five DOF system was to model a structure with a loose internal part. In order to detect this damage using the Holder Exponent, it is necessary to excite the structure with a periodic input. Furthermore, the study shows that sensors must be located within the proximity of the damage event.



**Figure 4.1.** Comparison of five DOF Holder Exponent signals at 10 Hz.

In the three-story structure, observations were made about the sensitivity of the Holder Exponent extracted from data collected at all the joints of the structure. Testing shows that discontinuities detected at the point of damage could be detected at a lower magnitude on the same floor of the structure. However, accelerometers away from the floor containing the damage event, even those on the same column, could not detect the loss of tension.

Unlike the five DOF structure, it is advantageous to have localized sensitivity to damage in the three-story structure. In a building, sensors can be placed at each joint and used to detect localized damage events during an earthquake. Results from the three-story structure show marginal intra-floor and poor inter-floor sensitivities of the Holder Exponent. In a real building, the attenuation of the damage event would be greater than the tested structure since the tested structure had a homogeneous aluminum floor. This sensitivity study shows that the Holder Exponent is effective for detecting localized damage.

## 5. CONCLUSIONS

After using the Holder Exponent to examine several different systems under varied excitation, several conclusions can be made about the applicability of using the Holder Exponent as a damage sensitive feature. For some excitations, such as periodic and low frequency random inputs, the Holder Exponent can be used very successfully to detect damage. However, the results from broadband excitations are rather poor. When the system is excited by a signal with a limited frequency content, the Holder Exponent reacted similarly to different damage events. In addition to excitation, factors such as location of instrumentation affect the sensitivity of the Holder Exponent. Results show the Holder Exponent to be more sensitive to the local response of the system, rather than the global response.

A statistical control process aimed at automatically detecting damage events was designed and subsequently implemented on data from each structure in the study. In some cases, the process was effective at flagging the damage events. However, some structural systems exhibited damage patterns in the Holder Exponent that were not detected by the statistical control process developed in this study. This lack of universal success shows that a statistical process control is generally application specific. However, because these damage patterns can be seen when visually inspecting the Holder Exponent data, there is hope that a statistical control process can be created that accurately detects these events.

Further research is recommended to quantify characteristics of the Holder Exponent. Using attributes such as the depth or width of a dip in the Holder Exponent, it may be possible to characterize the level or type of damage in the system. It is also recommended that efforts be made to quantify the sensitivity of the Holder Exponent to factors such as relative location of instrumentation and intensity of impacts. Methods of producing discontinuities not visible in the acceleration-time histories could prove very useful in further characterizing the potential of using the Holder Exponent in detection of structural damage.

The results of this study point to specific applications for the Holder Exponent as a damage sensitive feature. One of the most promising applications is the use of the Holder Exponent to detect damage during earthquakes, because the frequency of the ground motion of an earthquake is generally limited to between 0 and 10 Hz. This low frequency excitation to a structure is similar to the excitation used in the successful testing presented in this paper. Additional applications may present themselves as technology advances. For instance, the combination of sensors and actuators within a package will allow local excitation of the system with the desired waveform in order to accentuate the Holder Exponent's sensitivity to damage events. These and other applications can take advantage of incorporating the Holder Exponent in a damage detection scheme.

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